

Basic Integration Problems

I. Find the following integrals.

1. $\int(5x^2 - 8x + 5)dx$

2. $\int(-6x^3 + 9x^2 + 4x - 3)dx$

3. $\int(x^{\frac{3}{2}} + 2x + 3)dx$

4. $\int\left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3}\right)dx$

5. $\int\left(\sqrt{x} + \frac{1}{3\sqrt{x}}\right)dx$

6. $\int(12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx$

7. $\int\frac{x^2 + 4}{x^2}dx$

8. $\int\frac{1}{x\sqrt{x}}dx$

9. $\int(1 + 3t)t^2 dt$

10. $\int(2t^2 - 1)^2 dt$

11. $\int y^2 \sqrt[3]{y} dy$

12. $\int d\theta$

13. $\int 7 \sin(x) dx$

14. $\int 5 \cos(\theta) d\theta$

15. $\int 9 \sin(3x) dx$

16. $\int 12 \cos(4\theta) d\theta$

17. $\int 7 \cos(5x) dx$

18. $\int 4 \sin\left(\frac{x}{3}\right) dx$

19. $\int 4e^{-7x} dx$

20. $\int 9e^{\frac{x}{4}} dx$

21. $\int -5 \cos \pi x dx$

22. $\int -13e^{6t} dt$

II. Evaluate the following definite integrals.

1. $\int_1^4 (5x^2 - 8x + 5) dx$

2. $\int_1^9 (x^{\frac{3}{2}} + 2x + 3) dx$

3. $\int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}}) dx$

4. $\int_1^4 \frac{5}{x^3} dx$

5. $\int_{-1}^2 (1 + 3t)t^2 dt$

6. $\int_{-2}^1 (2t^2 - 1)^2 dt$

Solutions

I. Find the following integrals.

$$1. \int (5x^2 - 8x + 5) dx = \boxed{\frac{5x^3}{3} - 4x^2 + 5x + C}$$

$$2. \int (-6x^3 + 9x^2 + 4x - 3) dx = \boxed{\frac{-3x^4}{2} + 3x^3 + 2x^2 - 3x + C}$$

$$3. \int (x^{\frac{3}{2}} + 2x + 3) dx = \boxed{\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x + C}$$

$$4. \int \left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3} \right) dx = \int \left(\frac{8}{x} - 5x^{-2} + 6x^{-3} \right) dx$$
$$= 8\ln(x) - \frac{5x^{-1}}{-1} + \frac{6x^{-2}}{-2} = \boxed{8\ln(x) + \frac{5}{x} - \frac{3}{x^2} + C}$$

$$5. \int \left(\sqrt{x} + \frac{1}{3\sqrt{x}} \right) dx = \int \left(x^{\frac{1}{2}} + \frac{1}{3} x^{-\frac{1}{2}} \right) dx$$
$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{3} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{\frac{2}{3} x^{\frac{3}{2}} + \frac{2}{3} x^{\frac{1}{2}} + C}$$

$$6. \int (12x^{\frac{3}{4}} - 9x^{\frac{5}{3}}) dx = \boxed{\frac{48x^{\frac{7}{4}}}{7} - \frac{27x^{\frac{8}{3}}}{8} + c}$$

$$7. \int \frac{x^2 + 4}{x^2} dx = \int 1 + 4x^{-2} dx = \boxed{x - \frac{4}{x} + C}$$

$$8. \int \frac{1}{x\sqrt{x}} dx = \int x^{-\frac{3}{2}} dx = \boxed{-\frac{2}{\sqrt{x}} + C}$$

$$9. \int (1 + 3t)t^2 dt = \int t^2 + 3t^3 dt = \boxed{\frac{t^3}{3} + \frac{3t^4}{4} + C}$$

$$10. \int (2t^2 - 1)^2 dt = \int 4t^4 - 4t^2 + 1 dt = \boxed{\frac{4t^5}{5} - \frac{4t^3}{3} + t + C}$$

$$11. \int y^2 \sqrt[3]{y} dy = \int y^{\frac{7}{3}} dy = \frac{3y^{\frac{10}{3}}}{10} + C$$

$$12. \int d\theta = \theta + C$$

$$13. \int 7 \sin(x) dx = -7 \cos(x) + C$$

$$14. \int 5 \cos(\theta) d\theta = 5 \sin(\theta) + C$$

$$15. \int 9 \sin(3x) dx = -3 \cos(3x) + C$$

$$16. \int 12 \cos(4\theta) d\theta = 3 \sin 4\theta + C$$

$$17. \int 7 \cos(5x) dx = \frac{7 \sin(5x)}{5} + C$$

$$18. \int 4 \sin\left(\frac{x}{3}\right) dx = -12 \cos\left(\frac{x}{3}\right) + C$$

$$19. \int 4e^{-7x} dx = -\frac{4e^{-7x}}{7} + C$$

$$20. \int 9e^{\frac{x}{4}} dx = 36e^{\frac{x}{4}} + C$$

$$21. \int -5 \cos \pi x dx = -\frac{5 \sin(\pi x)}{\pi} + C$$

$$22. \int -13e^{6t} dt = -\frac{13e^{6t}}{6} + C$$

II. Evaluate the following definite integrals.

$$1. \int_1^4 (5x^2 - 8x + 5) dx = \left(\frac{5x^3}{3} - 4x^2 + 5x \right) \Big|_1^4 = \frac{188}{3} - \frac{8}{3} = \boxed{60}$$

$$2. \int_1^9 (x^{\frac{3}{2}} + 2x + 3) dx = \left(\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x \right) \Big|_1^9 = \frac{1026}{5} - \frac{22}{5} = \frac{1001}{5} = 200.2$$

$$3. \int_4^9 \left(\sqrt{x} + \frac{1}{3\sqrt{x}} \right) dx = \left(\frac{2}{3} x^{\frac{3}{2}} + \frac{2}{3} x^{\frac{1}{2}} \right) \Big|_4^9 = 20 - \frac{20}{3} = \frac{40}{3} = 13.333$$

$$4. \int_1^4 \frac{5}{x^3} dx = -\frac{5}{2x^2} \Big|_1^4 = -\frac{5}{32} + \frac{5}{2} = \frac{75}{32} = 2.344$$

$$5. \int_{-1}^2 (1 + 3t)t^2 dt = \left(\frac{t^3}{3} + \frac{3t^4}{4} \right) \Big|_{-1}^2 = \frac{44}{3} - \frac{5}{12} = \frac{57}{4} = 14.25$$

$$6. \int_{-2}^1 (2t^2 - 1)^2 dt = \left(\frac{4t^5}{5} - \frac{4t^3}{3} + t \right) \Big|_{-2}^1 = \frac{7}{15} + \frac{254}{15} = \frac{87}{5} = 17.4$$